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Flabio Dario Mirelez Delgado*

Master of Science
fmirelezd@ipn.mx
National Polytechnic Institute
Zacatecas, Mexico

Elena Elsa Bricio Barrios

Master of Science in Chemical Engineering
elena.bricio@colima.tecnm.mx
National Technological Institute of Mexico Colima Campus
Colima, Mexico

Santiago Arceo Díaz

Doctora en Educación Matemática
bvillarraga@unillanos.edu.co
Universidad de los Llanos
Villavicencio, Colombia.

***Autor para correspondencia:**
fmirelezd@ipn.mx

Fourier series in the planning and optimization of trajectories of autonomous aquatic vehicles

Abstract

The constant breeding of tilapia in ponds has emerged as a permanent source of income in rural areas, given the steady demand for this product. This work addresses the application of Fourier series and classical control in aquaculture, focusing on the integration of Fourier series for the parameterization of Cartesian trajectories guiding an autonomous vehicle. The vehicle aims to feed tilapia specimens (Oreochromis Niloticus) in an aquaculture tank. The goal is to minimize the construction cost of autonomous vehicle prototypes, ensuring safety through the calculation and correction of speed and acceleration to follow a predefined distribution route, promoting the growth of specimens. This is achieved through a control law based on a mathematical model. The use of parameterized Fourier series allows for efficient calculation of the vehicle's speed and acceleration, reducing the reliance on expensive sensors without compromising the safety of the autonomous vehicle.

Keywords: Fourier series, Mathematical modeling, Mobile robot, Smart aquaculture, Trajectory tracking.

Introduction

The improvement and optimization of industrial processes is one of the challenges facing developing nations. Likewise, the automation of these processes is a viable option to reduce downtime, improve the energy efficiency of the transformation of raw materials and provide a safe environment for process supervisors [1].

The miniaturization of electronic components and energy storage sources, essential for automation in industrial processes, has reduced the gap between theory and technological application. This has expanded the capacity to process information in real time for decision making, allowing the achievement of objectives established by the operator. The incorporation of advanced sensors, such as cameras or radars, for the collection of environmental data, together with the application of control and optimization algorithms, provides stability, precise trajectory design and effective decision-making capacity [2].

Autonomous vehicles are defined as those in which the need for a human operator has been eliminated. By incorporating these improvements, autonomous vehicles can perform tasks such as defusing explosives remotely and transporting people or goods to a predefined destination, drilling equipment in closed pit mines [3].

Over time, autonomous aquatic vehicles have emerged, specifically designed for navigation in bodies of water. Similar to their land counterparts, these vehicles have the ability to follow predefined trajectories, but unlike land vehicles, they must take into account the specific kinematics of the water body [4]. These devices have found significant applications, such as monitoring marine life at depth, identifying survivors in maritime accidents, and military surveillance in conflict zones [5].

Aquaculture is an industry that has seen significant improvements with the introduction of unmanned vehicles, such as water quality monitoring [6], real-time monitoring, transportation and distribution of food for marine fish based on cutting-edge technologies such as the Internet. of Things and Artificial Intelligence [7].

Although the implementation of these advanced systems can contribute to the sustainability of production processes, their acquisition cost is often prohibitive for most artisanal aquaculturists. Mexico, as a developing country, is in constant technical and technological updating and has focused on supporting primary productive sectors such as aquaculture and, in particular, the breeding and marketing of tilapia.

The breeding of these aquatic specimens, known as Oreochromis Niloticus, differs from other fish due to its resistance to diseases, its ease of reproduction and its adaptability to artificial environments [8]. This has led to the creation of aquaculture farms in 27 of the 32 states of Mexico with a production of more than 72 thousand tons of tilapia with an average cost of 5 dollars per kilogram [9].

Although this productive activity allows artisanal aquaculturists to be self-sufficient, one of the main challenges has been to establish methodologies that allow aquaculturists to obtain batches with the weight and size that can be marketed while reducing the cost of the feed that is estimated to be of around 30% of the fixed costs of the breeding process [10]. Although, feeding systems have been incorporated such as food sprinklers,

buoys anchored at strategic points in the breeding tank and scaffolding with buckets that release the food throughout the structure [11].
Having a mobile vehicle offers the advantage of programming a route that allows food to be dispersed in as extensive an area as possible, reducing problems such as aggressive competition between specimens to appropriate areas where excess food accumulates at the bottom of the pond. Since tilapias tend to consume food until it settles to the bottom, this strategy helps to avoid problems such as poisoning of specimens and the formation of harmful compounds associated with the decomposition of uneaten food, reducing the appearance of diseases.
This approach can reduce the investment cost of the microprocessor, sensor and microcontroller required to carry out a predefined route, thus addressing the problem from an analytical perspective. Therefore, this work proposes the design of power routes based on the Fourier series to calculate the speed and acceleration of the motors, minimizing the cost of developing these autonomous systems. Unlike other numerical strategies, Fourier series have been characterized by their ability to: i) decompose the trajectory into a series of sine and cosine functions, ii) create a smooth and safe trajectory if a control point is decided, and iii) allow calculating the speed and acceleration of the vehicle in real time for the incorporation of control schemes [12].

Materials and methods

Case study:

This proposal was developed in two stages: the identification of the needs of the aquaculturists at the study site and the computational development of a control algorithm that could be implemented in an autonomous aquatic vehicle.
In the first stage, data were collected corresponding to the tilapia breeding tanks in the "San Buenaventura" aquaculture farm, located in the Municipality of Armería, Colima. The identification of needs was developed through unstructured interviews with aquaculturists and aquaculture farm operators, where emphasis was placed on the need for an automated feeding system for tilapia in their juvenile and adult stages. These specimens are raised in artificial ponds in an area of 50 by 100 meters and a depth of 1.6 meters [13]. In these ponds it is common for support personnel to enter the body of water and disperse the feed by spraying the feed as it moves along the pond. However, the social dynamics of tilapia are based on the survival of the fittest, which is reflected in their aggressive behavior when feeding, preventing other tilapia from accessing food through attacks on the fins, causing the premature death of others. specimens and, in some cases, by not accessing food, the tilapia report weights below the average [8].
To improve the degree of food dispersion, a scheme was designed in which the vehicle moves to specific points and, after stopping, the food is released (see Figure 1). This proposal was reported by Rodríguez in [11] who proposed the design of an autonomous land vehicle that moved between beams that crossed the rearing pond.

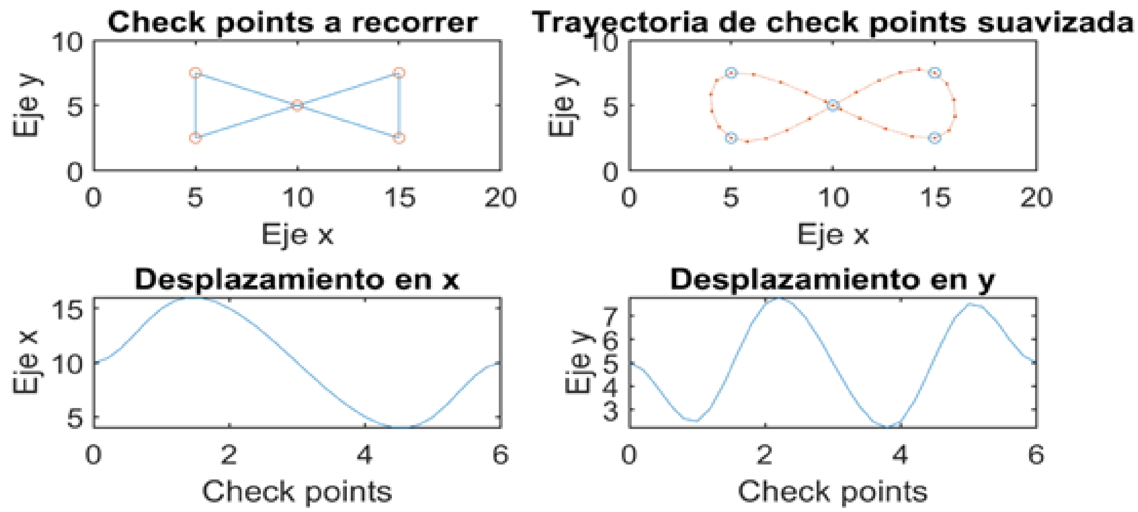

Figure 1. General representation for key point smoothing.

The second stage, was based on the definition and delimitation of the case study, was carried out in the facilities of the Tecnológico Nacional campus Colima and in the facilities of the National Polytechnic Institute. All computational development was coded in the MATLAB 2023b programming language.

The creation of an autonomous vehicle requires the application of the principles of mobile robotics. A classic example that can be used as a basis is the creation of a two-wheeled differential robot with a center of mass at the center of its axle and a passive wheel has a Cartesian configuration given by (see Figure 2), where are the generalized Cartesian coordinates, and are the coordinates of the center of mass of the robot and is the orientation angle of the robot, so its kinematic model can be expressed as shown in equation (1) reported [14].ξ=[x y θ]ξxyθ

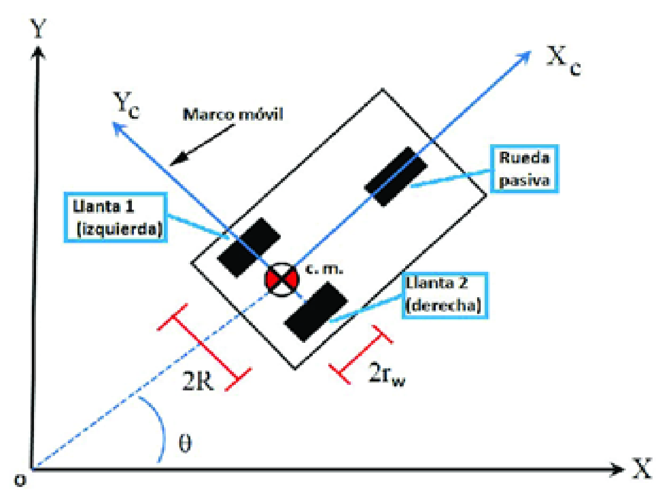

Figure 2. Representation of a mobile robot with wheels in the plane.XY

x˙=V cos(θ)
y˙=V sin(θ) (1)
θ˙= ω,

where is the linear speed of the mobile robot and ω is the angular speed, and the non-holonomic constraint (–) is also considered. The definition for both speeds is given by equations (2) and (3):Vx˙cos (θ)y˙cos (θ)=0

V= ((r_ω ψ˙_1+r_ω (ψ_2))˙)/2 (2)

ω= ((r_ω ψ˙_2-r_ω (ψ_1))˙)/2R, (3)

where and are the angular velocities of the left and right wheel respectively, is the wheel radius and corresponds to the distance between wheels through the axle that joins them. With the previous equations, a more complete kinematic model can be obtained, as shown in equation (4):ψ˙_1 ψ˙_2 r_ω 2R

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r_w \cos(\theta)}{2} & \frac{r_w \cos(\theta)}{2} \\ \frac{r_w \sin(\theta)}{2} & \frac{r_w \sin(\theta)}{2} \\ \frac{-r_w}{2R} & \frac{r_w}{2R} \end{bmatrix} \begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} \quad (4)$$

This representation is useful for a boat-type water vehicle, since the linear and angular velocities can be used as inputs for a control system or the angular velocities if the vehicle had wheels with blades on its sides. In any scenario it is feasible to modify the position of the device within a body of water using two control inputs.

Parametric function:

This design was reformulated by Hernández in [15], who proposed a homogeneous distribution of feeding at specific points in the breeding pond, taking into consideration the existence of aeration systems located near the center of the pond that fulfill the function of oxygenating the body of the fish. water, promoting suitable conditions for the growth of tilapia [16]. Although Hernández in [15] reports that the tracers are capable of describing a feeding route, the first and second derivative of the proposed parametric function diverges, that is, this system requires redefinition if this proposal is to be implemented in an aquatic vehicle. This work addressed this problem using Fourier series.

Trajectory planning by Fourier series:

In general, any closed trajectory can be represented mathematically by a periodic function, that is, a function that repeats itself at regular intervals of its independent variable, called "periods". However, in many cases the periodic functions that describe these trajectories may have characteristics that make the calculation of the first and second derivative difficult [17].

In this aspect, Fourier series play a crucial role in approximating periodic functions, since they allow a function to be efficiently decomposed into an infinite combination of sines and cosines. This representation significantly simplifies the calculation of the first and second derivative, by expressing the periodic function in terms of known trigonometric functions, whose derivatives consist only of sums of sines and cosines, facilitating mathematical analysis and optimizing calculations. By using Fourier series, a more manageable representation of the original function is achieved, which simplifies the differentiation process and improves analysis efficiency. The structure of a Fourier series is described below [17].

A function is considered periodic if there exists a positive number, such that for each value of the independent variable, , in the domain of it holds that:f(t)Ttf(t)

f(t)=f(t+nT), (5)

where is any non-zero integer and the number corresponds to the period of the function and represents the length of one complete repetition cycle.nT

Fourier series [18] propose that any periodic function can be represented as an infinite series:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{N} a_n cos(\frac{2\pi nt}{T}) + \sum_{n=1}^{N} b_n sin(\frac{2\pi nt}{T}), \quad (6)$$

where the sinusoidal functions represent harmonic oscillations with frequencies integer multiples of the fundamental frequency (proportional to). The coefficients1/T

$$a_0 = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)dt \quad (7)$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)cos(n\omega_0 t)dt \quad (8)$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)sin(n\omega_0 t)dt \quad (9)$$

determine the size of the contribution that each sinusoidal function has in the series and the fundamental frequency is defined as.ω_0=2ϖ/T

Results and discussion

Creation of tracking trajectories based on parametric equations:
Once we have the routes or trajectories that the device or vehicle must travel (see section "parametric function"), it was found that the proposed trajectory can be approximated by a Fourier series of five terms, as well as its transitive dynamics in the Figure 3.

$$x_d = 0.07247cos(6.423\frac{\pi\, t}{t_{max}}) - 0.3412\ sin(6.423\frac{\pi\, t}{t_{max}}) - 0.4004cos(2.141\frac{\pi\, t}{t_{max}})$$
$$-0.00969\ cos(4.282\frac{\pi\, t}{t_{max}}) + 5.731sin(2.141\frac{\pi\, t}{t_{max}})\ - 0.0690\ sin(4.282\frac{\pi\, t}{t_{max}}) + 10.0(10)$$

$$y_d = 0.5036cos(4.322\frac{\pi\, t}{t_{max}}) - 0.1248cos(8.643\frac{\pi\, t}{t_{max}}) - 0.03466cos(12.96\frac{\pi\, t}{t_{max}})$$
$$-2.496\ sin(4.322\frac{\pi\, t}{t_{max}}) + 0.2967sin(8.643\frac{\pi\, t}{t_{max}})\ +0.05096\ sin(12.96\frac{\pi\, t}{t_{max}}) + 5.0\quad \text{(eleven)}$$

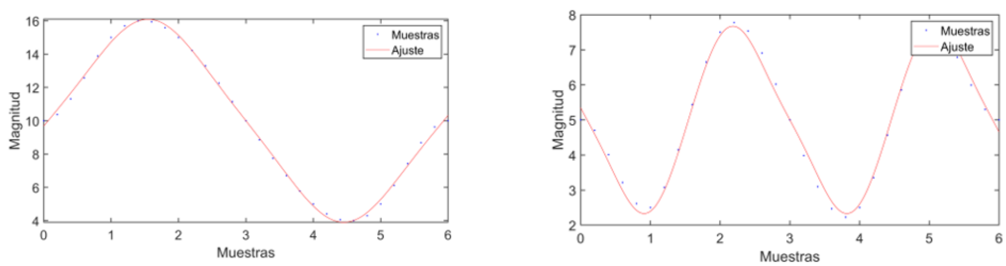

Figure 3. Panel a) and b) show, respectively, the comparison between the fit of the Fourier series (red line) and the check points (blue points).

After obtaining the parameterized trajectories as well as their derivatives, the next step is to select a control law that can be applied to the mobile robot to precisely follow the change of positions in the work area, that is, that the robot can stick to the route. To achieve the above, desired linear (12) and angular (13) speeds are established and, in this way, establish an error to be minimized between the real speeds of the prototype and the desired ones according to the trajectory to be fulfilled [19].

$$v_d(t) = \pm\sqrt{\dot{x}_d{}^2(t) + \dot{y}_d{}^2} \qquad (12)$$

$$\omega_d(t) = \frac{\ddot{y}_d(t)\dot{x}_d(t) - \ddot{x}_d(t)\dot{y}_d(t)}{\dot{x}_d{}^2(t) + \dot{y}_d{}^2(t)}. \qquad (13)$$

From the desired speeds that are a function of the desired positions, the control speeds or the control law can be established that allows position errors to be minimized and therefore follow the established routes [20], as can be seen in (14) and (15).

$$v = v_d cos(\theta_d - \theta) + k_1[cos\theta(x_d - x) + sin\theta(y_d - y)] \quad (14)$$

$$\omega = \omega_d + k_2 sign(v_d)[cos\theta(x_d - x) - sin\theta(y_d - y)] + k_3(\theta_d - \theta), \quad (15)$$

where:

$$k_1 = k_3 = 2\zeta\sqrt{\omega_d{}^2(t) + bv_d{}^2(t)}, \qquad k_2 = b|v_d(t)|, \ \zeta \in (0,1), \quad y \quad b > 0 (16)$$

The methodology used to simulate the behavior of a surface aquatic robot on a body of water, including the desired trajectories, the kinematic model and the control law, was built in Simulink. Figure 4 shows the general diagram in Simulink as well as the connections between the main blocks (path generator, control and kinematic model).

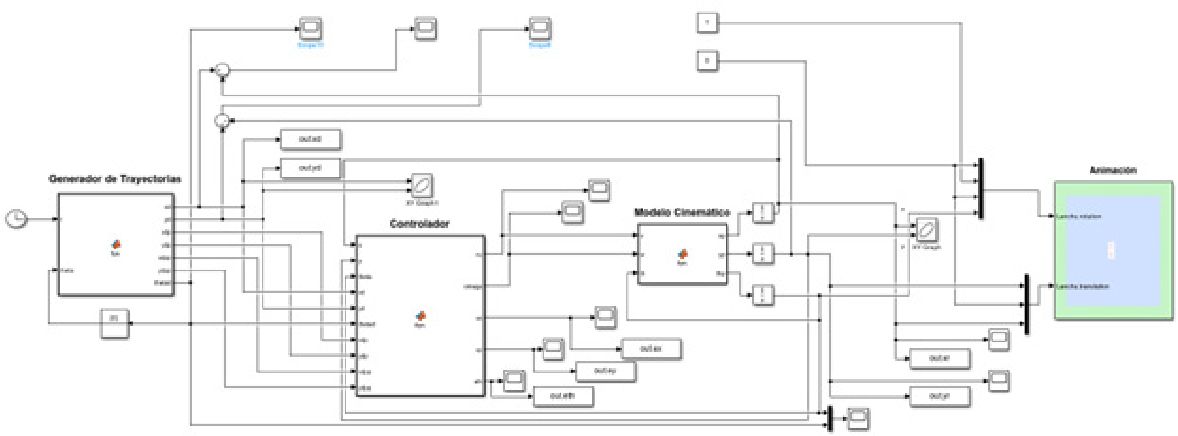

Figure 4. Simulink diagram for system simulation.

Following the methodology proposed by Chin in [2], a virtual environment generated in Simulink was built, an aquatic mobile robot was placed with initial conditions (10 m x 5 m) inside a simulated pond of 50 m x 50 m represented in Figure 5.

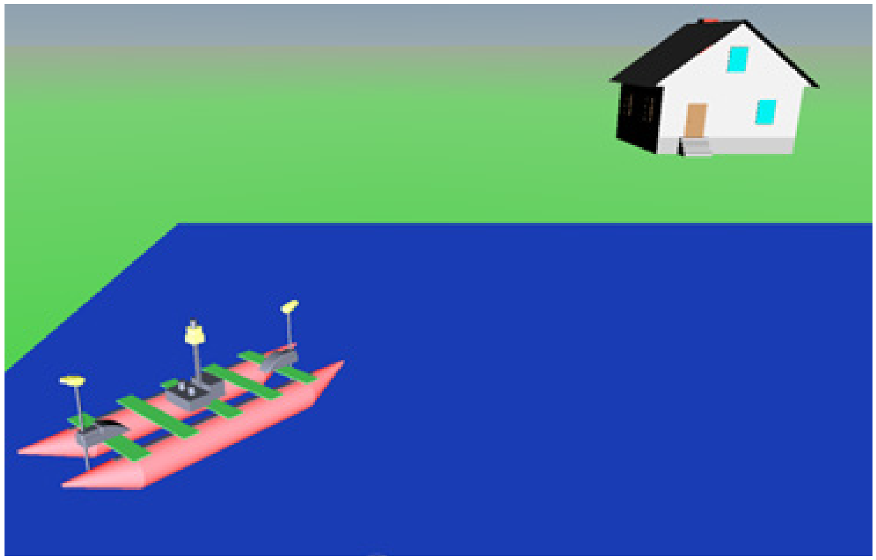

Figure 5. Virtual environment for the simulation of an aquatic vehicle.

Figure 6 reports the performance of the trajectory tracking in the x and y direction, as well as its errors in the orientation of the vehicle (). At first the magnitude of the errors is considerable but asymptotically they are minimized, although the error is not cancelled. This remains within a range considered acceptable.θ

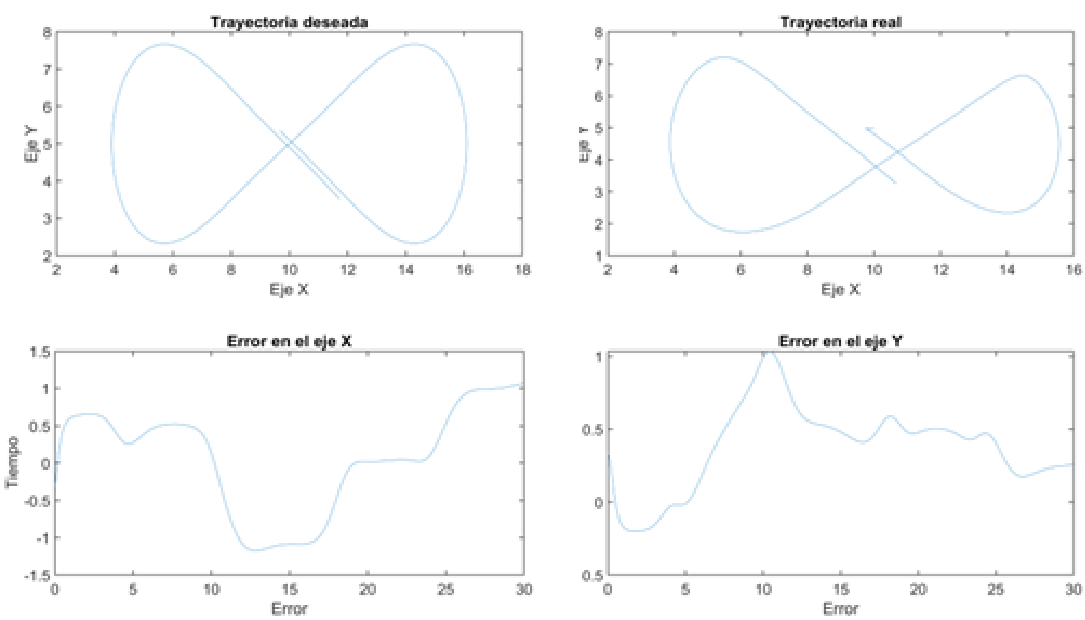

Figure 6: a) Desired trajectory adjusted with Fourier series, b) Simulated trajectory, c) Error in the x axis between the simulation and the desired trajectory, d) Error in the y axis between the simulation and the desired trajectory.

Conclusions

This work reports, from a numerical simulation perspective, the use of mathematical strategies to define, carry out and optimize food distribution routes inside a tilapia breeding tank, significantly reducing the cost of electronic components of an autonomous unmanned vehicle.
It is important to note that, for the selected parametric function (lenmiscata), it was necessary to couple a Fourier series to the tracers that would allow obtaining expressions of the first and second derivatives that are input variables in the kinematic and correction action model. The main advantage of using the Fourier series is that it can represent any function, therefore, any trajectory that you want to follow in the plane can have its first and second derivative and be applied to this proposal.
As future work, it is proposed to add natural disturbances such as air currents, irregular air flows generated by aeration systems and even the presence of native fauna in the area. Likewise, it is proposed to extend this proposal by allowing the vehicle to be programmed based on the growth stage of the specimen, which will allow better nutrition and thus gain in weight and size.

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